



Mathematics Department

Math 330

Final Exam

2<sup>nd</sup> Semester 20/21

Student name: ..... ID no.: .....

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**Circle your final Answer, You should show how you get the answer, we will only grade the supported answer, Each problem worth 3points**

1) Using four digits arithmetic and rounding, Find the value of

$$\frac{7}{17} + \frac{81}{13} + \frac{801}{19}$$

**Answer=**

2) When using the bisection method to estimate the solution of the equation  $f(x) = 0$  on the interval  $[4,6]$ , find the number of iterations needed to get accuracy  $10^{-5}$ .

**Answer=**

3) Use the secant method with  $p_0 = 1$ ,  $p_1 = 1.5$  to estimate the solution of the equation  $x^5 = x + 4$ , Find the next iteration.

**Answer=**

(4) Find the repulsive fixed point of  $g(x) = \frac{10}{x} + 3$

**Answer=**

5) Find the order of convergence of the following sequence of numbers that converges to  $p=1$ , Prove your answer numerically

$$p_0 = 1.2000000000$$

$$p_1 = 1.006060606$$

$$p_2 = 1.000006087$$

$$p_3 = 1.000000000$$

**Answer=**

6) When estimating the roots of the function  $f(x) = (x + 3)^3(x - 1)$  using Newton Method, find the asymptotic error constant  $A$  for  $p = 1$

**Answer=**

- 7) Find the point on the parabola  $y = x^3$  that is closest to the point  $(1, 2)$  with two digits accuracy of the  $x$  coordinate.

**Answer=**

- 8) Using a table, Find  $f$  [1.3, 2.4, 3.6] where  $f(x) = x^2$

**Answer=**

- 9) Find  $L_{3,2}(5)$  using the nodes

$$x_0 = 3, x_1 = 4, x_2 = 6, x_3 = 8$$

**Answer=**

10) Find the cost of evaluating  $p_2(x)$ , for a specific  $x$ , where  $p_2(x)$  is the Lagrange interpolating polynomial

**Answer=**

11) Find the best upper bound for the error when using Newton polynomial  $p_3(x)$  to estimate  $f(x) = \ln(x + 1)$  in the interval  $[0.1, 0.4]$  and using uniform partition.

**Answer=**

12) If the following is a cubic spline over  $[0, 2]$

$$S(x) = \left\{ \begin{array}{ll} -2x^3 + 2x^2 + ax + 1, & 0 \leq x \leq 1 \\ 7(x-1)^3 - 4(x-1)^2 + b(x-1) + 1, & 1 < x \leq 2 \end{array} \right\}$$

Find  $a$  and  $b$

**Answer=**

13)- Consider the following formula

$$f''(x_0) = \frac{f_3 - 4f_0 + 3f_{-1}}{6h^2} - \frac{2hf'''(c)}{3}$$

Find the optimal  $h$

**Answer=**

(14) Approximate  $\int_{-1}^1 x^2 e^{x^2} dx$   
Using Simpson's rule

**Answer=**

15)- Estimate  $f'(4)$ , and  $f''(4)$  using central difference formulas of order  $o(h^2)$  for the data (0,1), (2,4), (4,6), (6,9)

**Answer=**

16)- Consider the quadrature formula

$$\int_{-6}^6 f(x)dx \cong Af(-6) + Bf(6)$$

If the degree of precession is 1, Find  $A, B$

**Answer=**

17) - Consider the quadrature formula

$$\int_{-1}^1 f(x)dx \cong \frac{4}{5}f\left(-\frac{1}{2}\right) + \frac{6}{5}f\left(\frac{1}{3}\right)$$

If the degree of precession is 1, Find the truncation error.

**Answer=**

Good Luck